Analyzing Cooperative Lane Change Models for Connected Vehicles

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Abstract— **This paper examines intelligent lane change models based on the cooperation among connected vehicles for traffic management and travel time optimization. Lane change decisions and speed controls could be coordinated and optimized to reduce the overall braking and achieve greater traffic throughput. In an effort to design a distributed cooperative lane change assistant (D-CLCA) within the European Commission's project REDUCTION*, this paper describes the requirements associated with an optimal lane change behavior, and evaluate existing lane change models based on these requirements. These models are evaluated for travel times, fuel consumption, number of lane changes and the overall braking globally for all vehicles on the considered road segment. We have developed traffic simulations using different traffic densities for both symmetric and asymmetric lane changes and different levels of cooperation among vehicles. Our empirical analysis shows that an optimal lane change model should optimize the conflicting requirements of maintaining desired speeds and reducing the number of lane changes and fuel consumption for all vehicles simultaneously. These results will be used to develop an intelligent distributed and cooperative lane change assistant.**

I. INTRODUCTION

With BMW going to test their hands free automated lane change technology in early 2015, and Audi and Mercedes-Benz putting efforts to introduce this feature in their Traffic Jam Assistant for stop-and-go traffic, analyzing and modeling lane change maneuvers has attracted focus of ITS researchers in last few years. Moreover, recently researchers in traffic management have empirically shown that lane change maneuvers are primarily responsible for most of the traffic perturbations on multilane freeways [1],[2]. These perturbations ultimately end up in traffic congestion causing delays in travel time and extra fuel [4]. Aggressive lane changes on highways or city-type traffic, result in increased accel/decel which burns 20-30% extra fuel [24], and consequently more CO2 emissions. Such maneuvers are also critical to driving safety as 13,784 accidents (with casualties) in Germany happened in a lane change maneuver [3].

Developing an intelligent lane change assistant with the aim to increase traffic throughput is a challenging task as it requires planning at the level of microscopic traffic variables, heterogeneous road conditions with varying levels of drivers' perceptions and behaviors. Recently, researchers have proposed lane change prediction methods based on trajectory tracking to alarm the driver of the subject vehicle about the safety of a lane change maneuver [5], [6]. Currently on the market are Hella lane change warning system [7], a blind spot detection system by Mobileye [8], and almost all car manufacturers now offer lane keeping systems in their particular models. These systems only serve the security aspect of a lane change maneuver of the individual vehicle. Although secure lane changing is crucial to safe driving, maintaining desired speeds and reducing frequent lane changes are important aspects of driving comfort. These factors affect not only the individual driver, but also carry a significant impact on the capacity, stability and breakdown of traffic flows. Efficient traffic management under high traffic volumes, requires a cumulative optimal action plan for all vehicles.

Our hypothesis is that lane change decisions and speed controls could be coordinated and optimized to reduce the overall braking and achieve greater traffic throughput. Connected vehicles using Vehicle-to-Vehicle (V2V) communication and distributed learning algorithms, can search for optimal local lane change and car following plans in such a way that it not only helps them maintaining their own desired speeds but also does not affect the speeds of other vehicles negatively. Besides safety criterion, lane change and car following models should be evaluated in terms of reduced travel times, incurred number of lane changes, fuel consumption and resulting CO2 emissions.

As a first effort towards the realization of distributed cooperative lane change assistant (D-CLCA) system for connected vehicles, within the European Commission's project REDUCTION*, in this paper we analyze and evaluate state-of-the-art cooperative lane change model MOBIL(Minimizing Overall Braking Induced by Lane Changes)[14], with respect to the common driver behaviors during lane changing maneuvers on motorways. MOBIL is a cooperative lane change model for microscopic car-following models and allows lane changes only if they increase the sum of accelerations of all the involved vehicles i.e. the current vehicle and its old and new followers. The reason to choose MOBIL for our analysis is that it is the only lane change model which considers the effect of lane changes on the stability of traffic flow in the near vicinity. The model results in locally optimal decisions but its global effect on the whole traffic needs to be evaluated. On the other hand, common lane changing behaviors of drivers on multi-lane freeways, as studied by German Aerospace Center (DLR)[9] is implemented using

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SUMO's lane change model[11]. This study showed that although it is common to make lane changes to drive faster, drivers also show some sort of cooperation by giving lanes to faster following vehicles.

To analyze the effect of cooperation among connected vehicles during lane change maneuvers, simulations are developed for the traffic on a 3-lane highway with different densities (vehicles/hour/lane). Vehicles communicate through standard DSRC protocols to coordinate their lane change behavior. The results of these experiments show that frequent lane changes towards faster lanes reduce the overall travel time of the involved vehicles, but at the cost of safety, fuel and driving discomfort. On the other hand, delayed lane change decisions and simply giving way to the fast vehicles, increase the overall deceleration. An optimal trade-off between these two extreme behaviors needs to be explored by planning lane change trajectories ahead in time, through coordination and distributed optimization.

The work in this paper is exploratory in nature, and provides a basis to develop distributed cooperative lane change assistance system. Our approach of analysis is different in the sense that we focus on optimizing the cumulative effects of lane change maneuvers on the traffic flow management. The rest of this paper is organized as follows: Section 2 describes related work. Section 3 introduces lane change models used in this paper and describes requirements associated with optimal lane change behavior. Our simulation set-up and experimental evaluations are described in Section 4, and the last section concludes the paper.

II. RELATED WORK

Frequent or arbitrary lane changing maneuvers based on individual preferences and decisions have been noted as a cause for traffic congestion, the appearance of bottleneck phenomena or broadly speaking to the creation of unfavorable road and driving conditions. Towards this direction many studies have provided a wealth of insights and solutions. Rule-based approaches include those reported in [12][13]. From their perspective lane changes may occur on varying "gap-acceptance" conditions, lane density and traffic speed. A decision to make a lane change is further classified based on different driving incentives (obligation or preference). Mandatory lane changes are considered those which occur because of a blocked lane, traffic regulations or in order to follow one's route to destination. Discretionary changes are made in order for the subject vehicle to achieve better lane conditions i.e. higher speed or moving to a lane with lower density. The interpretation of these approaches however results in complex models with many parameters and thus lead in diverse lane changing behaviors. Toledo et al. in [18] proposed a model that jointly evaluates mandatory and discretionary lane changes and later in [19] an explicit target lane model was studied were the lane with the highest utility is elected as a destination lane. In [16] the authors studied the lane changing decisions of aggressive drivers, i.e fast vehicles frequently changing lanes on a highway, and their impact to the overall traffic flow. An approach similar to MOBIL is adopted in [17] by scheduling the lane change maneuvers of autonomous connected vehicles.

III. COOPERATIVE LANE CHANGE MODELS

In this section, we first briefly describe MOBIL as a cooperative lane change model, followed by the description of common driver behaviors on highways and their implementation through the lane change model of SUMO. We then present a lane change objective function which describes the requirements associated with optimal lane change behavior for increasing the traffic flow.

A. Minimizing Overall Braking Induced by Lane Changes (MOBIL)

MOBIL [14] is a general lane change model based on microscopic longitudinal accelerations which determine the incentive and risk associated with a lane change decision. Incentives based lane changing typically focuses on improving the traffic situation of an individual driver by letting him drive faster or avoid a slow leader. On the other hand, while considering a lane change decision, MOBIL's incentive criterion also considers immediately affected neighbors. For symmetric traffic, the following condition allows a lane change only if this increases the sum of accelerations of all the involved vehicles i.e. the current vehicle and its old and new followers.

$$\widetilde{a}_c + p\,[\widetilde{a}_n - a_n + \widetilde{a}_o - a_o] > \Delta a_{th} \tag{1}$$

where a_c is the acceleration of vehicle c in the current lane and $\widetilde{a}_c$ is its acceleration after the prospective lane change. Similarly current and consequent acceleration of vehicle n (the successor of c in the target lane) and vehicle o (follower of c in the current lane) can also be estimated. p is a politeness parameter which controls the degree of cooperation while considering a lane change, from a purely egoistic behavior ($p = 0$) to an altruistic one ($p \geq 1$). It balances the deceleration of other vehicles with the gain in its own acceleration. Given a value of threshold Δa_{th}, MOBIL prevents lane changes if the overall advantage is only marginal as compared to keeping the current lane.

For asymmetric traffic rules of European motorways, where right lane is the default lane and overtaking through this lane is prohibited, MOBIL modifies the symmetric rule (1) by adding a bias (Δa_{bias}) to threshold Δa_{th} when considering a lane change from right-to-left, whereas for a left-to-right decision the bias is subtracted in order to implement the keep-right directive. To prevent vehicles from right-overtaking, it influences the acceleration in the right-lane such that the advantage of changing to the right lane for overtaking is minimal.

Longitudinal accelerations ($\widetilde{a}$, a) in (1) are determined using microscopic car-following models such as Intelligent Driver Model (IDM) [25], Gipps model [12] or the velocity difference model [26]. In our experiments, we have used IDM acceleration a^* defined in terms of ratio of the current velocity v to the desired velocity v_{pref}, the ratio of current bumper-to-bumper gap s to the desired gap s^* and the relative velocity difference Δv from the leading vehicle.

$$a^* = a\left[1 - \left(\frac{v}{v_{pref}}\right) - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right] \tag{2}$$

where the desired gap s^* is computed as in (3). Here s_0 is the minimum gap, b is the comfortable deceleration and T is time gap. The first term $s_0 + vT$ determines bumper-to-bumper safe distance, and the second term $\frac{v\Delta v}{2\sqrt{ab}}$ defines an intelligent braking strategy.

$$s^*(v, \Delta v) = s_0 + max\left(0, vT + \frac{v\Delta v}{2\sqrt{ab}}\right) \tag{3}$$

For more details on MOBIL and IDM, readers are referred to [27].

B. Common Cooperative Lane Changing Behaviors

In this section, we first refer to a recent study by the German Aerospace Center (DLR) [9] to briefly describe common cooperative lane changing behaviors of human drivers on motorways. We then use the lane change model of SUMO [11] to implement these behavioral rules as a baseline method.

To investigate the cooperative behavior of drivers during lane changing maneuvers, DLR performed a multi-driver simulation study. 20 participant drivers, from the DLR pool of 800 test persons, were asked to drive in a multi-driver simulator developed by DLR. This simulator allows participants to drive within the same simulation scenarios. For more details on the simulator, please refer to [10]. A scenario in Figure-1 is considered where a vehicle V_1 driving in the right lane, is blocked by a braking leader V_2, and considers to change to middle lane to maintain its speed. For this lane change decision, behaviors of drivers are examined, if a faster follower V_3 is approaching in the target lane (middle lane). Depending upon the strength of V_2's braking (hard/weak), and availability of free left lane to V_3, most of the drivers of V_1 in this test study, exhibited the following lane changing behaviors:

- If left lane is available to V_3, most drivers of V_1 decided to change lane and merged in-front of V_3, and hence requested a cooperation. Most V_3 drivers changed to the left lane expressing cooperation to V_1.
- Otherwise, if left lane is blocked, most V_1 drivers waited for V_3 to overtake (merge behind V_3).
- Drivers of V_1 more often changed the lane when V_2 made a hard braking.

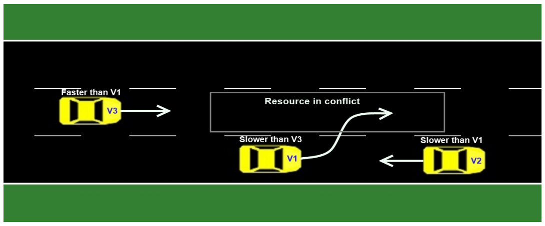


Fig. 1: A lane change conflict scenario (Adapted from [9])

We summarize this expected behavior as; most drivers tend to change lanes for driving faster or maintaining their speeds. But they also exhibit cooperative behavior by changing lanes for faster followers or by restraining from lane changes to let the faster followers overtake first. We denote this common behavior as cooperative speed gain (CoSpG).

Next we present how the lane change model of SUMO [11] can be used to implement CoSpG behavior.

1) Lane Change Model of SUMO: This model computes valid lanes through the network along the route of each vehicle. A lane is valid if it can be used for continuing the route without requiring to change the lane. For this purpose, each vehicle using its current speed v and vehicle length l, computes the distance (s_{safe}) to the obstructing leader, after which the route cannot be continued. A vehicle must change the lane if this distance is less than the required distance needed for a lane change.

$$s_{safe} = v \times \alpha + 2l \tag{4}$$

For discretionary lane changes - for driving fast - the model of SUMO computes a benefit of changing the lane. The benefit is simply the net gain in instantaneous speed on the target lane $v(t, ln)$ at time t as compared to current speed $v(t, lc)$ on the current lane. Again, this speed is computed using a car-following model and is normalized by the maximum velocity v_{max} of vehicle under free-flow conditions.

$$b_{ln}(t) = \frac{(v(t, ln) - v(t, lc))}{v_{max}} \tag{5}$$

If the benefit of changing the lane is greater than a certain threshold (in either direction depending upon the sign of benefit), the vehicle changes the lane. This rule can be used to represent the common speed gain behavior of drivers.

In case the vehicle is blocked by a leader or a successor in the target lane, it starts to interact with the blocking vehicle and all the involved vehicles make adjustments to their speed to let the requesting vehicle change to the target lane. These speed adjustments are made as per the following rules:

$$\begin{aligned} v_{accel}(t) &= v_{cf}(t) + \frac{v_{max}(t)}{2} \\ v_{decel}(t) &= v_{cf}(t) + \frac{v_{min}(t)}{2} \end{aligned} \tag{6}$$

where $v_{cf}(t)$ is the car-following speed in m/s, $v_{accel}(t)$ is the new speed of a vehicle if it is blocking or is blocked at its own back, and $v_{decel}(t)$ is the new speed of the vehicle if it is blocking or is blocked at its own front. These rules can be used to implement the common cooperative behavior of driver as mentioned earlier.

C. Optimal Lane Change Control for Traffic Management

Most of the existing work in lane change modeling takes a car-centered approach to predict lane change safety and letting individual vehicles to drive faster. Contrary to this, we study the global influence of lane change models on the overall traffic (in a busy road segment or motorway) i.e. *How to optimize a coordinated lane change control among* *connected vehicles to increase the traffic flow (reduced braking), reduce travel times and fuel/CO2 emissions and still letting all vehicles to drive at their preferred or close to preferred speeds?*

In this study we analyze lane change models which consider cooperation among drivers during lane change maneuvers. Particularly, we evaluate MOBIL and common lane change behavior of Cooperative Speed Gain (CoSpG) based on the following characteristic metrics of traffic flow management.

- **Delay**: Aggressive and careless lane changes induce braking perturbations which travel against the flow of traffic and depending upon the traffic density can increase the travel times considerably. For a traffic of n vehicles, (7) defines the delay. Here T_i denotes actual total travel time of vehicle i, T_i^{pref} is the desired travel time. v_i and v_i^{pref} refer to actual and desired speeds, respectively. L is the total length of road.

$$delay = \sum_{i=1}^{n} \left(T_i - T_i^{pref}\right) \qquad \text{where } T_i = \frac{L}{v_i},\ T_i^{pref} = \frac{L}{v_i^{pref}} \tag{7}$$

- **Lane Change Rate**: Frequent and large number of lane changes not only affect the traffic flows negatively but also adds to driving discomfort. An optimal lane change model is required to minimize this metric. (8) defines it as a function of average traffic density (Vehicles/Lane/Hour). Here, N_i is the number of lane changes for vehicle i, per kilometer (Δx) of road , and for each time interval (Δt).

$$lcRate(d) = \sum_{i=1}^{n} \frac{N_i}{\Delta x \times \Delta t} \tag{8}$$

- **Consumption and CO2 Emissions**: We found it very interesting to analyze the effect of lane change behaviors on the total fuel consumption and CO_2 emissions of the overall traffic. In order to predict traffic emissions accurately and with more spatial and temporal details, we have used EMIT model [20], which is an engine load based model and explains the physical phenomena, that generates emissions, very well. Fuel consumption is mainly dependent on the engine speed and the engine power. For space limitation, we do not provide the formulas for fuel and CO_2 estimation, but can be accessed from [20].
- **Deceleration**: An optimal lane change model is required to decrease the overall deceleration which significantly affects the delays and fuel consumption. (9) describes the overall braking induced in the traffic.

$$Braking = \sum_{i=1}^{n} (\tilde{a}_i - a_i < 0) \tag{9}$$

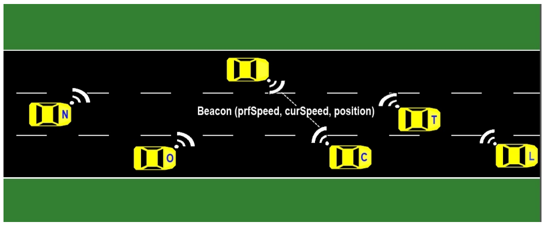


Fig. 2: Exchange of vicinity attributes using beacons

With the aim to define a distributed lane change control which optimizes the above mentioned metrics, in (10) we have proposed an objective function which combines the key independent metrics in a single equation.

$$\begin{aligned} Objective = &\sum_{i=1}^{n} reg_t \left(T_i - T_i^{pref}\right) + \alpha \sum_{i=1}^{n} \frac{N_i}{\Delta x \times \Delta t} \\ &+ \beta \sum_{i=1}^{n} reg_t \left(\tilde{a}_i - a_i < 0\right) \end{aligned} \tag{10}$$

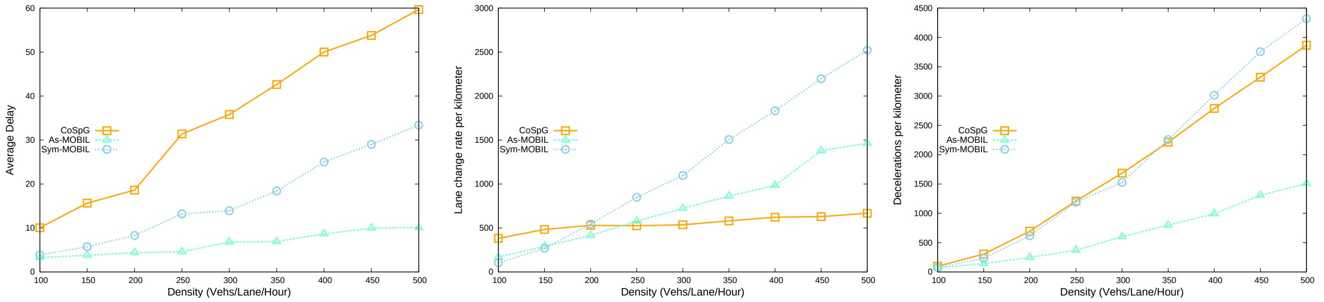
where reg_t is an instantaneous regret function and can be defined as $reg_t(\Delta T) = max(0, \Delta T)$ (eg. for travel time metric T). α and β are the penalty weights which determine the relative priority of each metric. Coordinated optimal lane change control should minimize this function to help increasing the traffic flow.

IV. SIMULATION AND EVALUATIONS

In this section, we first describe the simulation set-up, model parameters, vehicle attributes and road segment specification. Later, we present our simulation results with respect to different characteristic metrics described in section III for various traffic densities.

A. Simualtion Environment

For our experimentation we used the open source framework for vehicular simulations, VEINS[21], built on two standard simulators, the network simulator OMNET++ [22] and the road traffic simulator SUMO [23]. Our evaluation is conducted on a 10 kilometers highway road comprised of 3 lanes where each lane is assigned a speed limit of 50*m/s*. Heterogeneity affects the lane changing behavior and thus upon entering the simulation each vehicle is assigned with a preferred speed uniformly distributed in the range of 20-50(m/s). With such configuration vehicles obtain different objectives, perform different lane changes and thus exhibit different driving behaviors. Each vehicle is introduced in the simulation with the minimum allowed speed through our experimentation i.e. 20*m/s*. For the density of the scenarios we introduce vehicles at each separate lane with a rate of 100 to 500 vehicles per lane per hour. For the CO2 emissions we use the EMIT model [20] for Category-9-vehicle e.g., Dodge Spirit 1994. Following on MOBIL's configuration each vehicle is set with acceleration of $1.5m/s^2$ and deceleration at $2m/s^2$. Vehicles length is set at $4m$ whereas the minimum gap between them is configured at $2m$. Communication between vehicles is established through DSRC with a range of 500m. Beacon messages are utilized in order for a vehicle to be aware of its vicinity attributes (current speed, preferred speed, distance, etc.) used for MOBIL's lane changing decisions. Figure-2 depicts our implementation approach. While



(a) (b) (c)

Fig. 3: (a) Average delay incurred for each traffic density [t-test: *p-value*=0.007]; (b) Lane change rate [*p-value*$<$0.001]; (c) Overall braking induced by lane changes [*p-value*$<$0.001]

considering a lane change decision, current vehicle C uses the vicinity attributes received from the current leader L, leader in the target lane T, the current follower O and the new follower N. The simulation update step is set to 0.1 seconds.

B. Evaluations

As mentioned above, we have performed experiments with different traffic densities (Vehicles/Hour/Lane) to evaluate MOBIL for both symmetric (Sym-MOBIL) and asymmetric (As-MOBIL) traffic and common lane change behavior of cooperative speed gain (CoSpG). Evaluation metrics include average delay, total braking, number of lane changes, fuel consumption, CO2 emissions, average time of driving at preferred speed, the effect of different levels of cooperation and performance analysis with a percentage of vehicles using MOBIL and the others exhibiting common lane change behavior.

Figure-3a shows the average delay incurred using three types of lane changing approaches, for different traffic densities. CoSpG lane changing results in significantly larger delays as compared to MOBIL. This can be explained with a reason that MOBIL only considers those lane changes which increase the cumulative acceleration of the immediate neighboring vehicles. At the same time, depending upon threshold Δa_{th}, with a slight increase in the cumulative acceleration, it lets vehicles make lane changes and drive faster. This results in large number of lane changes as compared to CoSpG as shown in Figure-3b, and reduced overall braking (Figure-3c). On the other hand, CoSpG vehicles do consider lane changes for maintaining their own speeds, but they also bear braking to show cooperation to faster vehicles or delay the lane change decision to the point when route cannot be continued further. As-MOBIL keeps the right directive and overtaking is only performed from left lane. Due to this control, it incurs much reduced delay, number of lane changes and overall braking as compared to all-lanes freedom of Sym-MOBIL. Moreover, the performance of each model deteriorates with the increase in traffic density.

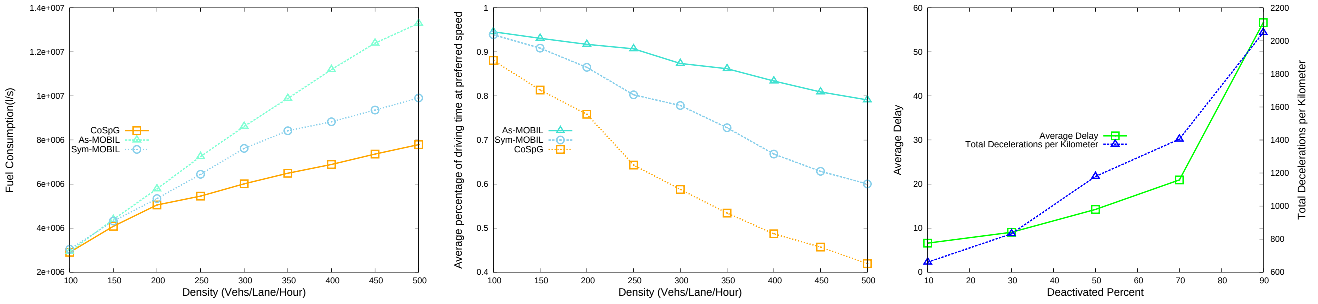
Based on the above results, Figure-4a compares the fuel efficiency of the lane change models. Since As-Symetric lets the vehicles drive faster with much reduced overall braking and delays, it incurs large fuel consumption as compared to Sym-MOBIL and CoSpG. Another interesting result is depicted in Figure-4b. It shows that As-MOBIL lets the vehicles drive at their preferred speed for most of their driving time. This endorses the earlier results of As-MOBIL with reduced delay and overall braking. It is important to analyze the system performance within the mix of cooperative/smart vehicles and the legacy vehicles. This can be represented by the percentage of vehicles not using MOBIL i.e. they are using the default lane change behavior CoSpG.

Plot in Figure-4c shows that as the percentage of vehicles not using MOBIL increases, the delay and overall braking increases as well. Lastly, we analyze the effect of lane change threshold and politeness parameter on the performance of MOBIL, in terms of considerable cumulative gain and heterogeneous cooperative behaviors. Figure-5 depicts that at $\Delta a_{th} = 0$, MOBIL results in large number of lane changes, which drop significantly for $\Delta a_{th} > 0$, and then see a gradual decrease with increasing thresholds. Also politeness at $pol = 0$ represents an aggressive behavior with a lot of lane changes as compared to cooperative behavior represent by $pol = 1$.

V. DISCUSSIONS AND CONCLUSIONS

In this paper we have analyzed the effect of coordination among communicating vehicles during lane change maneuvers, on the traffic flow management. Our approach of analysis is different in the sense that we focus on the cumulative effects of lane changing maneuvers such as overall traffic delays, braking, fuel/CO2 emissions and the number of lane changes. As per our hypothesis, these traffic effects can be minimized if the involved vehicles coordinate their lane changing and car-following actions according to a distributed intelligent plan. Particularly, in this exploratory study we evaluate cooperative lane change model MOBIL with respect to common lane change behaviors of drivers. We chose MOBIL as it is the only lane changing model which takes into account the effect of lane change decisions on the immediate neighbors. Based on the simplistic control rules, this model partially relates to our idea of increasing the overall traffic flow. On the other hand, it was more appropriate to analyze the affects of usual lane change behaviors of drivers on the overall traffic.

In our simulated evaluations, we found that MOBIL keeps its promise of reducing overall braking in the European



(a) (b) (c)

Fig. 4: (a) Fuel Consumption [t-test: *p-value*<0.001]; (b) Driving duration at preferred speed [*p-value*=0.02]; (c) Performance with the percentage of vehicles not using MOBIL

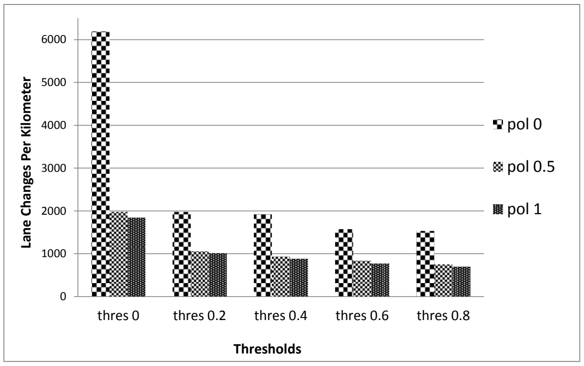


Fig. 5: Effect of threshold and politeness

type asymmetric traffic. It lets the vehicles drive at their desired speeds for a larger time of their travel duration, in comparison to vehicles exhibiting usual cooperative speed gain behavior. But this performance lift comes at the cost of making large number of frequent lane changes and burning much more fuel with CO2 emissions. One reason for this is that, at each time step, MOBIL vehicles keep checking for a slight (depending upon Δa_{th}) cumulative advantage in the local vicinity considering immediate followers. With common lane change behaviors, drivers only react to the local situation when it occurs. Lane is changed only if it is not possible to continue the route using the current lane.

We can conclude that both approaches do not plan ahead or are unable to predict the road situation in the next 500 meters or 1 kilometer. We are using the findings of this paper in our next work on developing a distributed cooperative lane change assistant (D-CLCA). For this we intend to use V2V communication in a bit larger road neighborhood such that vehicles can coordinate to search for their optimal trajectories ahead in time, in such as way that optimizes the objective function in (10).

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